FULL EMPLOYMENT POLICY AND ECONOMIC GROWTH

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Within the context of a simple aggregative model, this paper explores how the rate of labor force growth and the rate of technical advance influence the fiscal and monetary policies needed to maintain equality of aggregate demand and potential output. The model attempts to tie together two strands of analysis that have developed in the literature—a neoclassical strand stemming from Solow’s work on the determinants of growth of potential output and a neo-Keynesian strand stemming from the Harrod-Domar studies of the conditions under which full employment can be maintained in a growing economy [8] [9] [4] [1].

I. The Basic Model

The framework of the model is traditional. The supply of output at full employment (or potential output) is assumed to be a Cobb-Douglas function of capital and labor, given the level of technology. To simplify notation in subsequent calculations, and to facilitate generalization in the appendix, the production function will be written in a slightly unconventional form:

\[ Q = K^\alpha (L \cdot A)^{1-\alpha}, \]

where \( Q \) is potential output, \( K \) is the stock of capital, \( L \) is the labor supply and \( A \) an index of labor effectiveness. Technical progress is assumed to be purely labor augmenting. (In the Cobb-Douglas production function, labor augmenting technical progress and Hicks neutral technical progress are equivalent.) Thus the rate of growth of potential output is:

\[ \frac{\dot{Q}}{Q} = \frac{\dot{K}}{K} + \alpha \frac{\dot{A}}{A} + (1 - \alpha) \frac{\dot{L}}{L}. \]

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1 There are a number of other treatments of this same general topic, the relationship between growth of potential output and the conditions required for full employment. Tobin [10] [11], Smith [7], Eisner [2], and Hall [3], in particular, have dealt with this question, but within the context of models that differ from the present one in significant respects.

2 The more conventional formulation is:

\[ Q = B K^\alpha L^{1-\alpha}, \]

and

\[ \frac{\dot{Q}}{Q} = \frac{\dot{B}}{B} + \alpha \frac{\dot{K}}{K} + (1 - \alpha) \frac{\dot{L}}{L}. \]

If \( A = B^{\text{1/1-\alpha}} \), the two formulations, of course, are exactly equivalent.
\[ \frac{Q}{Q} = \alpha \frac{K}{K} + (1 - \alpha) \left( \frac{L}{L} + \frac{A}{A} \right). \]

In the appendix to this paper a more general production function is used. While there are a number of important added complications, it is demonstrated there that many of the implications of the model employing a Cobb-Douglas production function carry over, with qualifications, when a more general neoclassical production function is employed, so long as it is assumed that technical progress is purely labor augmenting.

On the demand side, I ignore exports and imports and assume total effective demand is given by:

\[ Y = C + G + I, \]

where \( Y \) is total demand and \( C, G, \) and \( I \) are consumption, government spending, and investment, respectively. Consumption is assumed to be a constant fraction of disposable income which in turn is equal to GNP minus taxes at a constant proportional tax rate, \( t \). There are no business retained earnings or depreciation allowances. Government spending is a constant fraction, \( g \), of GNP. Thus:

\[ Y = c(Y - tY) + gY + I. \]

Defining \( s = 1 - c \), this can be written in the familiar form:

\[ Y = \frac{I}{s + t(1 - s) - g}. \]

The concept of "full employment" in equation (1) is a normative one relating to the socially optimal degree of labor utilization. In general the measured rate of unemployment will not be zero at "full employment" output, nor is this the maximum output that can be attained; however, if output is pushed (or pulled by demand) beyond this level there will be stronger inflationary pressures. Full employment output thus represents output at the degree of labor utilization the society deems optimal, balancing the benefits in terms of decreased unemployment and greater output that result as demand is increased, against the greater inflationary tendencies and, perhaps, higher unit costs, that also are a concomitant of greater pressure of demand on a given supply of labor and capital.\(^{2a}\)

\(^{2a}\) The relationship between output and increased labor force utilization need not, and in general will not, follow equation (1). One would expect average labor quality to decline as labor utilization rates increased. While in a situation where the economy was operating with capital being underutilized the decreased labor quality effect might be for a while be offset by more efficient operation of existing capital, we are concerned with sustained degrees of labor utilization, and it can be assumed that capital adjusts to that rate. Thus in the long-run production function with the (sustained) degree of labor utilization as a variable, output should be positively related to amount of employment but negative related to the degree of labor force utilization.
A principal objective of fiscal and monetary policy, then, is to adjust actual output so that it is equal to full employment output, neither lower with higher unemployment rates, nor higher with more rapid price increase. For this objective to be achieved, the demand for goods and supply of goods at full employment, \( Y \) and \( \bar{Q} \), must be equal over time.\(^3\)

If we focus on a moment of time, say time \( t_0 \), then the level of effective demand generated by investment through the multiplier will equal full employment output when:

\[
I_0 = [s + i(1 - s) - g]Q_0.
\]

For full employment to persist into the next moment of time:

\[
\left( \frac{I}{I} \right)_0 = \left( \frac{Q}{\bar{Q}} \right)_0.
\]

These conditions of course do not imply that full employment will persist further. But under certain conditions \( I/I \) and \( \bar{Q}/\bar{Q} \) will be constants. If they are constants and if equations (6) and (7) hold initially, \( Y \) and \( \bar{Q} \) will continue to be equal.

The key relationship in the model is the investment equation. Investment is both an important contributor to growth of potential output, and the principal dynamic determinant of the level of aggregate demand. I assume that the level of investment is determined by the volume of profitable opportunities to expand the capital stock. In this paper we are examining conditions that must hold for full employment (and full utilization of the capital stock) to be realized; if there is no slack, there will be profitable opportunities for new investment if the marginal productivity of capital (at full employment of labor) exceeds the interest rate.\(^4\) Indeed, at interest rate \( r \) there will be profitable investment opportunities so long as the actual capital stock falls short of \( K^* \), the profit maximizing stock, where:\(^5\)

\[^3\text{I assume that, if the goods market is in balance at full employment, the labor market is also—that is, that the real wage rate is such that the demand for labor equals the full employment supply of labor when aggregate demand and supply of output are in balance.}\]

\[^4\text{There are, of course, two major qualifications to this. First, the correct statement of the conditions for expected profitability is that the expected discounted flow of future net returns exceeds the cost of new equipment. This involves a look into the future. The condition of excess of short-run marginal productivity over the existing interest rate is a proxy for the correct profitability conditions only under rather special circumstances. Second, under conditions of risk aversion or imperfect capital markets, the cutoff on investment decisions might occur at expected rates of return significantly higher than the interest rate.}\]

\[^5\text{\( K^* \) is derived by solving for \( K \) in the equation:}\]

\[
r = \frac{\delta \theta}{\delta K} = \alpha K^{(\alpha - 1)}L^{(1 - \alpha)}A^{(1 - \alpha)}.
\]

Again, it should be noted that I am assuming the wage rate is compatible with full employment.
\[ K^* = \left( \frac{\alpha}{r} \right)^{1/(1-\alpha)} L \cdot A. \]

The investment equation is:
\[ I = \lambda (K^* - K), \]
or
\[ I = \lambda \left[ \left( \frac{\alpha}{r} \right)^{1/(1-\alpha)} L \cdot A - K \right] \]

Thus the greater the opportunities for profitable expansion of the capital stock, the greater the rate of investment.

The dynamics of the model are provided by labor force growth and technical advance. Notice that these factors have two effects. First, both increase potential output. In the popular discussion of the problem of maintaining full employment it seems to be generally believed that the more rapid are technical progress and labor force growth, the greater is the difficulty in achieving an equal growth of demand. But, of course, a second effect of technical progress and labor force growth is to increase the size of the capital stock that is profitable at any given interest rate—and thus to spur investment and increase effective demand. (A third effect, then, is to further spur potential output through the increase in investment they stimulate.) If we take $A/A$ and $L/L$ as autonomous, and are free to vary $r$ and $t$, under what values of $r$ and $t$ will the capacity-creating and demand-increasing effects of technical progress and labor force growth be equal? Is it true that if labor force growth or technical progress is more rapid, a more expansionary government policy is needed? These are the questions explored in the following sections.

Although these are Harrod-Domar type questions, the model presented here enables explicit recognition that the growth of the labor force and the advance of technology affect both the natural rate of growth (the growth of potential output) and the growth of investment; in the Harrod-Domar analysis the effect of these variables on investment demand is suppressed. The underlying model of growth of potential output follows Solow’s neoclassical model. However, Solow does not work with explicit demand equations and hence cannot examine the conditions of sustained full employment; obviously this hinges on the

\[
\begin{align*}
\delta_0 &= W \\
\delta L &\mid L = L \text{ full} \\
K &= K^*
\end{align*}
\]
assumed response of investors and savers to various conditions. Thus this model is a marriage of the two strands.

Before proceeding with the analysis it is important to make clear one important thing that the model does not try to do. The model says nothing about what happens if the dynamic equilibrium conditions are not met—if there is either excess demand or involuntary unemployment. There are two themes in the Harrod-type analysis—a statement of conditions of dynamic equilibrium, and a statement about what happens if there is excess capacity. The simple investment function used in this model could be enriched to take account of slack and of unfilled orders for new equipment, but I have not done this in the formal model. Also, the model contains no mechanism, explicit or implicit, to explain why the interest rate is what it is; it only attempts to say what the interest rate must be to be compatible with full employment, given the other variables of the system.⁶

II. Full Employment Without Fiscal Policy

Let us begin by assuming no government spending or taxes. Assume also that technical progress and labor force growth are proceeding at constant rates. The variable that is free is the interest rate. What is required of the interest rate if full employment is to be achieved and maintained?

Recalling the necessary conditions [equations (6) and (7)], the investment demand equation (9a), and the growth of potential output equation (2), the requirements for full employment to exist and persist into the next moment of time are:

\[(6a) \quad I(A, L, r, K) = sQ\]

\[(7a) \quad \frac{I_A A + I_L L + I_r r + I_K K}{I} = \alpha \frac{\dot{K}}{K} + (1 - \alpha) \left( \frac{L}{L} + \frac{A}{A} \right).\]

If we ignore depreciation (which is treated in the appendix), the two equations are, of course, linked by the identity: \(I = \dot{K}\). The relevant calculations are made in a footnote.⁷

⁶ Notice that the behavior equations of the model do not involve any wealth or financial variables explicitly. The model could be enriched to include these, as in the Tobin and Hall models. However, the benefits of the simple treatment used here are that certain important relationships stand out more strongly. Implicitly, something like the following is assumed with respect to financial and wealth variables. The allocation of accumulated consumer saving between money and claims on real assets is a function of the interest rate; however, the saving rate is not a function of either the interest rate or of wealth. The managers of business firms invest in new plant and equipment and pay for this by selling claims. The government finances its deficits by selling bonds whose rate of return is pegged to the rate of return on claims on real assets. The money price level is rigid.

⁷ The more explicit form of (6a) is:
One implication is that, under our assumptions, full employment will be compatible with a constant interest rate only if conditions are such that the capital stock and potential output are growing at the same rate. Or:

\[
\frac{\dot{K}}{K} = \frac{Q}{\dot{Q}} = \frac{L}{\dot{L}} + \frac{A}{A}.
\]

Since \(\dot{K}/K = sQ/K\) (assuming full employment), the saving rate, the output-capital ratio, the growth of labor force, and the growth of technology must be related as follows:

\[
\frac{Q}{K} = \frac{1}{s} \left( \frac{L}{\dot{L}} + \frac{A}{A} \right).
\]

This "golden age" result was to be expected. Under the production function assumptions the rate of return on capital will rise or fall depending on whether the output-capital ratio is rising or falling. If \(Q/K\) initially falls short of (11), at investment equal to full employment saving, \(\dot{K}/K\) will fall short of \(\dot{Q}/Q\) and hence both the output-capital ratio and the rate of return on capital will rise. If \(Q/K\) exceeds (11), investment equal to full employment will lead to a fall in the output-capital ratio and the rate of return on capital. Under our investment assumption, such a rise or fall will lead to an increase or decrease in investment as a fraction of potential GNP, unless offset by a compensating change in the interest rate. Since saving is assumed to be a constant percentage of GNP, these compensating changes in \(r\) are then required for maintenance of equality between \(\dot{Y}\) and \(\dot{Q}\).

The system will asymptotically approach the golden age solution if \(\dot{L}/L, \dot{A}/A, \text{and } s\) remain constant. Let us now examine what this asym-
totonally required interest rate is, and the variables on which it depends. Setting \( \dot{r} = 0 \) and \( Q/K = 1/s(\dot{L}/L + \dot{A}/A) \) in equations (6a) and (7a) yields the following expression for the full employment interest rate in golden age equilibrium:

\[
\dot{r} = \frac{\alpha}{s} \left( \frac{L}{A} + \frac{A}{A} \right) \left[ \frac{\lambda}{\dot{L}/L + \frac{\dot{A}}{A} + \lambda} \right]^{1-\alpha}.
\]

(12)

If \( \lambda \) is very large the condition stated in equation (12) approaches:

\[
\dot{r} = \frac{\alpha}{s} \left( \frac{L}{A} + \frac{A}{A} \right).
\]

(12a)

For some purposes (12a) will be easier and more illuminating to work with.\(^9\)

If the \( \dot{r} \) of equation (12) prevails, the growth of potential output generated by \( \dot{L}/L \) and \( \dot{A}/A \), and the growth of effective demand spurred by the impact of \( \dot{L}/L \) and \( \dot{A}/A \) on investment opportunities, will be in balance. Equation (12) can be interpreted as defining Wicksell's natural rate of interest—the interest rate at which investment demand and full employment saving are equal. Equation (12) also can be interpreted as defining the conditions under which the natural rate of growth will equal the warranted rate of growth in a Harrod-like formulation of the problem; if we work with (12a), \( \dot{L}/L + \dot{A}/A \) clearly is the natural rate of

\(^8\) Manipulation of the Cobb-Douglas equation allows (6b) to be written:

\[
\lambda \left[ \left( \frac{\alpha}{r} \frac{Q}{K} \right)^{1/1-\alpha} K - K \right] = sQ.
\]

(6c)

Dividing by \( K \) and noting equation (11) and that \( \dot{K}/K = sQ/K \) we have the following triplet of equalities which define the conditions of sustained full employment in golden age equilibrium:

\[
\frac{L}{A} + \frac{\dot{A}}{\dot{A}} = \lambda \left[ \left( \frac{\alpha}{r} \frac{Q}{K} \right)^{1/1-\alpha} - 1 \right] = sQ/K.
\]

Solving the first two for \( Q/K \):

\[
\frac{Q}{K} = \left( \frac{\dot{L} + \frac{\dot{A}}{A} + \lambda}{\lambda \left( \frac{\alpha}{r} \right)^{1/1-\alpha}} \right)^{1-\alpha}.
\]

Noting that \( Q/K \) also must equal \( 1/s(\dot{L}/L + \dot{A}/A) \) and solving for \( r \) yields equation (12).

\(^9\) Of course (12a) could have been derived directly by assuming that the actual capital stock always equaled the profit-maximizing capital stock—there are never any unexploited investment opportunities—and working with an investment equation, \( I = K^* \). Tobin, in his 1964 AER paper [11], derives this equation by examining what will be the marginal productivity of capital in the golden age. Of course, this will be the full employment compatible interest rate only if it is assumed that \( K = K^* \).
growth, equation (8) shows that the desired capital-output ratio,
\((K^*/Q)\), equals \(\alpha/r\), and thus \(s(r/\alpha)\) is the warranted rate of growth. If
the actual and natural rates of interest are equal, then there will be
equality of the natural and warranted rates of growth. An interest rate
enters this formulation but not Harrod's because of the difference in
investment functions. In this formulation, unlike Harrod's, there are
a number of capital-output ratios that will satisfy businessmen, not just
one, because the desired capital-output ratio is a function of the interest
rate.\(^{10}\) If the interest rate is fixed and not a variable, the requirements
for sustained full employment of this model are identical to Harrod's,
despite a somewhat different investment demand equation and produc-
tion function.\(^{11}\)

Equation (12) shows that given \(\dot{L}/L\) and \(\dot{A}/A\), the interest rate com-
patible with full employment is negatively related to the saving rate.
To balance a higher saving rate, investment must tap opportunities
with a lower rate of return. Given the saving rate, the interest rate will
be higher the faster the growth of the labor force, and the more rapid
technological progress. While this last result may be surprising, the
reason is obvious. Technological advance and labor force growth are the
key factors determining the rate at which profitable investment oppor-
tunities grow and a given investment rate will yield a higher sustained
rate of return when they proceed rapidly than when they are slow. If we
treat \(r\) as fixed and \(s\) as a variable to be adjusted to match the invest-
ment-GNP ratio, then the result of faster technological progress or
labor force growth would be an increase in the investment-GNP ratio
(and a required increase in the saving rate), as equation (13) shows:

\[
\frac{I}{Y} = \frac{\alpha}{r} \left( \frac{L}{L} + \frac{A}{A} \right) \left[ \frac{\lambda}{\frac{\dot{L}}{L} + \frac{A}{A} + \lambda} \right]^{1-\alpha}.
\]

This is a result clearly in the spirit of Schumpeter. But if the saving rate
is fixed, with faster technical progress or labor force growth the interest
rate must be increased to keep investment from outrunning full employ-
ment saving.

Thus, in this model at least, given any existing interest rate, an ac-
celeration of technical progress or labor force growth provides even
more of a spur to demand than to potential output. In a world where the

\(^{10}\) This, of course, has been the main bone of contention between Solow [19] and Tobin [8],
on the one hand, and Harrod [4] on the other.

\(^{11}\) Eisner makes the point that the Harrod rigidity of (or a ceiling on) the capital-output
ratio can stem from either rigidity of the desired capital-output ratio, or from interest rate
rigidity or an interest rate floor. If the elasticity of substitution were less than one, then a given
change in the desired capital-output ratio would require a far larger change in the interest rate,
and the floor interest rate might be hit at a quite low capital-output ratio.
interest rate adjusts up and down to equilibrate investment and full employment saving, this result would be of no particular significance. But if the interest rate is sticky, an acceleration of these forces will be reflected in decreased unemployment or inflationary pressures.

III. The Requirements for Stabilization Policy

Active stabilization policy may be required for two different reasons. First, in the absence of active policy it may be impossible to achieve the parameter values consistent with full employment; this might be the case, for example, if there is an interest rate floor. Second, even if the parameters can be achieved without active policy, tendencies of the system to move and stay at full employment may be weak. The concern about full employment generated by the early Harrod-Domar models involved both aspects—the required conditions appeared infeasible (in the absence of government deficit spending programs to reduce total saving as a fraction of GNP) and, in addition, full employment seemed a highly unstable equilibrium.

As pointed out earlier, the model above contains no analysis of the stability of the full employment path, nor will any formal analysis be presented here. However, the following brief discussion may help to put the problem in context.

Assume that when aggregate demand and potential output are not equal, the difference first shows up in unfilled orders or unintended inventory accumulation. Let us focus on the latter case of deficiency in aggregate demand. Assume that there are two kinds of reactions taken by business firms. First, they reduce investment expenditures; thus the basic investment equation must be modified by adding a term reflecting underutilization of existing capital. Second, they reduce production and income payments. They may or may not lay off workers or cut back on new hires—if they do not, they reduce hours and, in any case, incomes are reduced in line with cutbacks in output. Consumption then will fall.

Thus in this model, there may be present the type of instability discussed by Harrod despite strictly neoclassical investment and production assumptions. The sensitivity of investment and output decisions to the accumulation of inventories is a strong destabilizing force. To the extent that the interest rate rises or falls depending on the difference between the existing level of investment and full employment saving, this is a stabilizing force. However, one would expect that, even if the interest rate adjusts relatively quickly, the short-term sensitivity of the desired capital-output ratio to the interest rate might be much smaller than the long-term sensitivity as given by equation (7).\(^{12}\)

\(^{12}\) In the limit with no short-run sensitivity of the desired capital-output ratio to the interest
The extent to which active stabilization policy is required depends on
the balance between the destabilizing effects of accumulating inven-
tories and growing underutilization of capacity, and the stabilizing
effects of interest rate adjustments. In the following discussion it will be
assumed that conscious policy actions are required at least from time to
time to keep aggregate demand and potential output in balance.

Let us now introduce the government sector and government stabili-
ization policy into the model. If the income tax rate is \( t \), and govern-
ment expenditures as a fraction of GNP are \( g \), then the conditions of full
employment in the steady state are:\(^{13}\)

\[
(12b) \quad r = \left( \frac{\alpha}{s + t(1 - s) - g} \right) \left( \frac{L}{L} + \frac{A}{A} \right) \left[ \frac{\lambda}{\hat{L}/L + \hat{A}/A + \lambda} \right]^{1 - \alpha},
\]

(12a) becomes

\[
(12c) \quad r = \left( \frac{\alpha}{s + t(1 - s) - g} \right) \left( \frac{L}{L} + \frac{A}{A} \right).
\]

For given values of \( s, g, \hat{L}/L, \hat{A}/A, \) and \( \lambda \), this equation enables us to
trace out the alternative pairs of \( r \) and \( t \) that are consistent with full
employment in a golden age.\(^{14}\) The curve is a rectangular hyperbola,
as shown in Figure 1, which will shift to the right or left depending on
the strength of the other factors that determine demand.

Note that, with fiscal policy now admissible, the previous results
regarding the interest rate compatible with full employment are gen-
eralized as follows: the faster are technical progress or labor force
growth, the tighter must be stabilization policy. This statement applies
to the steady state in which \( \hat{K}/K = \hat{Q}/Q \) and a constant \( r \) and \( t \) are com-

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\(^{13}\) The results here are very similar to those of Smith [7] and of Musgrave [5]. However, the
investment equation in this model is explicitly dynamic.
patible with sustained full employment. Assume that we are on one long-run equilibrium time path and that, suddenly, the rates of technical progress or the rate of growth of the labor force rises. What is required of policy?

One possibility is to shift immediately to the new steady state path through fiscal policy. In the new steady state $\dot{K}/K$ must rise to equal the now increased $\dot{L}/L+\dot{A}/A$; thus room must be made for a higher ratio of investment to GNP through increased taxes so that 

$$[s+\dot{t}(1-s)-\dot{g}]Q/K = \dot{L}/L+\dot{A}/A$$

at the new higher level. If $\lambda$ is infinite, investment will rise to the needed higher fraction of potential GNP at the old interest rate. If $\lambda$ is not infinite, equation (12b) shows that some reduction in the interest rate will be required. Once these changes in the tax and interest rates are made, the system is on a golden age path again, with a higher investment-GNP ratio.

Another possibility is to stress monetary policy and to respond more gradually. Taxes can be maintained and the interest rate tightened to keep investment demand from rising as a fraction of potential GNP. With technical advance now more rapid, and the interest rate increased to keep investment at the same fraction of GNP, the rate of growth of the capital stock will be faster than it was because GNP will be growing faster. But it will not immediately rise to equal the faster rate of growth of potential output. Therefore the capital-output ratio will gradually fall toward its new (lower) steady state equilibrium. And as it falls, the
rising marginal productivity of capital will require further increases in 
the interest rate. Eventually the new equilibrium will be reached.

The fact that these choices are open to the government means that full 
employment policy and growth policy cannot be separated; the target 
degree of labor utilization can be achieved through different alternative 
policies which have different effects on present consumption and future 
output. The golden rule for golden age advocates would keep the invest-
ment—GNP ratio constant and equal to capital’s share of GNP, and 
vary the interest rate up or down following variations in \( \dot{L}/L + \dot{A}/A \). 
Those who believe in a constant social rate of time preference would 
keep \( r \) constant and at that rate, and permit \( I/GNP \) to rise or fall with 
\( \dot{L}/L + \dot{A}/A \), varying the tax rate to make room.

Returning to the steady state conditions, an interesting question is: 
under what conditions will full employment be consistent with a govern-
ment surplus? For simplicity we will use equation (12c). Rearranging:

\[
t - g = \frac{\alpha}{r} \left( \frac{L}{L} + \frac{A}{A} \right) - s + st.
\]

Letting \( S^* = t - g \) be the government surplus as a fraction of potential GNP:

\[
S^* = \frac{\alpha}{r} \left( \frac{L}{L} + \frac{A}{A} \right) - s + sg
\]

(14)

Equation (14) shows some familiar results. The balanced budget mul-
tiplier requires the surplus at full employment without inflation to be 
larger (or the deficit smaller) as government spending increases as a 
function of potential GNP. And obviously the surplus must be larger, 
or the deficit smaller, the looser is monetary policy. The new result is 
that, given \( g \) and \( r \), the faster technical progress and labor force growth, 
the greater will be the surplus or the smaller the deficit needed to keep 
\( Q \) and \( V \) in line.\(^{15}\) In a very real sense the higher tax rates required when 
technological change is rapid should be viewed as the price we have to 
pay to achieve rapid growth without inflation. But if there are pressures 
against increasing government spending as a percentage of GNP, if 
there exist strong political pressures against running sustained budget 
deficits, and if there is a floor to the interest rate, rapid technical pro-
gress, far from being a threat to full employment, may be a prerequisite 
for it.

\(^{15}\) Here the ignoring of the effect of financial wealth and liquidity is important. Obviously 
the size and sign of the deficit has an effect on the economy through these variables as well as 
through disposable income.
Appendix

If we assume that the capital stock depreciates at rate \( \delta \), this affects the basic equations in two ways. First, at the profit maximizing capital stock, the marginal productivity of capital is equated to \( r + \delta \), not \( r \). Second, the rate of growth of the capital stock is equal to \( sQ/K - \delta \), not \( sQ/K \). This affects the conclusions in the following ways. First, the golden age output-capital ratio is

\[
\frac{Q}{K} = \frac{1}{s} \left( \frac{L}{L} + \frac{A}{A} + \delta \right).
\]

Second, the golden age interest rate is

\[
r = \frac{\alpha}{s} \left( \frac{L}{L} + \frac{A}{A} + \delta \right) \left[ \frac{\lambda}{\frac{\dot{L}}{L} + \frac{\dot{A}}{A} + \lambda + \delta} \right]^{1 - \alpha} - \delta.
\]

Thus the qualitative conclusions change in no essential way.

Relaxing the Cobb-Douglas assumption raises a number of complications. In particular Hicks neutral technical progress no longer is necessarily equivalent to labor augmenting technical advance, and further there may be upper or lower bounds on the output-capital ratio which preclude equality of the rate of growth of the capital stock and the "augmented" labor force for certain values of \( s, \dot{L}/L, \) and \( \dot{A}/A \). However, assuming technical progress is labor augmenting and that a golden age (with a positive elasticity of output with respect to labor) is possible within the domain of interest, the qualitative results apply to a model with a more general neoclassical production function.

The production function can be written in the more general form

\[
Q = F(K, L \cdot A).
\]

Assuming technical progress is labor augmenting:

\[
\frac{Q}{Q} = \left[ \frac{F_k K}{K} \right] \frac{\dot{K}}{K} + \left[ \frac{F_{LA} L \cdot A}{Q} \right] \left( \frac{L}{L} + \frac{A}{A} \right).
\]

If \( F(\cdot) \) has the standard neoclassical properties and there is perfect competition, the terms in brackets will be the shares of capital and labor, respectively. Investment will be profitable up to the point where \( F_k(K, L \cdot A) = r \). Under our assumptions this is equivalent to \( F_k(K/L \cdot A, 1) = r \). The profit-maximizing capital stock is \( K^* = F_k^{-1}(r) \cdot L \cdot A \). Notice that the forms of these more general equations are very similar to the Cobb-Douglas special cases.

The conclusions of the model would appear to depend strongly on the existence of a stable golden age equilibrium where \( sQ/K = \dot{L}/L + \dot{A}/A \). With a Cobb-Douglas function \( Q/K \) increases without finite upper bound as \( L \cdot A/K \) increases, and declines toward zero as \( K/L \cdot A \) increases; thus there is a \( Q/K \) which can satisfy the golden age equation for any positive...
values of \( s, \frac{\dot{L}}{L}, \) and \( \dot{A}/A. \) This does not apply generally; for certain neo-
classical production functions \( Q/K \) may be bounded above as \( L \cdot A/K \)
increases, or approach a positive lower bound as \( K/L \cdot A \) increases.

So long as we are considering values of \( 1/s(\dot{L}/L+\dot{A}/A) \) which do not fall
outside the bounds of \( Q/K, \) the qualitative conclusions regarding the relation-
ship of \( \dot{L}+\dot{A}/A \) to the full employment interest rate carry over to
this more general formulation. Within this range at least, the marginal pro-
ductivity of capital is an increasing function of \( Q/K = h(L \cdot A/K), \) and the
higher the marginal productivity of capital the higher must be the interest
rate required to keep investment demand equal to full employment saving
(given the investment and saving assumptions of the model). This result
does not depend on the sign or size of the elasticity of substitution, although
the quantitative sensitivity of \( r \) to \( \dot{L}/L \) and \( \dot{A}/A \) will depend on the exact
shape of the isoquant. Further, it is interesting to note that, for the case
where the elasticity of substitution is less than one, labor's share will tend
to be smaller in the full employment equilibrium when technical change
and labor force growth are rapid than when they are slow; the opposite
holds if the elasticity of substitution exceeds one.

However, if the elasticity of substitution is less than one, it is possible
that at very rapid rates of growth of labor or technical advance, it will be
impossible to achieve a golden age equilibrium with capital and "aug-
mented" labor growing at the same rate. For as \( Q/K \) and \( L \cdot A/K \) increase,
the elasticity of output with respect to labor will fall, and it may fall to zero
before \( Q/K \) can equal \( \dot{L}/L+\dot{A}/A/s. \) In this case asymptotically \( \dot{Q}/Q \) and
\( \dot{K}/K \) will grow at the same pace but slower than \( \dot{L}/L+\dot{A}/A. \) A related
problem happens when the elasticity of substitution is greater than one, and
\( \dot{L}/L+\dot{A}/A \) is small. (For an excellent discussion, see Pitchford [6].)

If \( s \) and \( \dot{L}/L+\dot{A}/A \) take on values such that either of these conditions
occurs, the analysis becomes quite complicated, depending on the exact
form of the production function. Historically, however, we have not come
close to experiencing situations where labor's share of national income was
driven close to zero. If the rest of the assumptions of the model do not
strain reality too much, the limitation that the model only applies to a par-
ticular range of \( s, \frac{\dot{L}}{L}, \) and \( \dot{A}/A \) [the range depending on \( F(\cdot) \)] does not
appear to detract from any heuristic value it might have.

References

2. Robert Eisner, "On Growth Models and the Neo-Classical Re-
6. J. D. Pitchford, "Growth and the Elasticity of Substitution," *Econ.